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Тезисы решений задач теоретического тура
Theoretical round. Sketches for solutions

язык / language	English

Group A,B

1. Let us compare the light fluxes observed by our "naked eye" from Sputnik I (in zenith at perigee) and from the full Moon.
   If W = light flux from the Sun near Earth,
   α_* = albedo of the observed objects,
   $S_* = \pi r_*^2 = \pi d_*^2/4$, where r_* and d_* – radius and diameter of the observed objects,
   R_* = distances from the observed objects to the observer:
   Flux from Moon to observer: $I_M = W\cdot\alpha_M\cdot S_M / (2\pi R_{M\text{-}O}^2)$.
   Flux from Sputnik I to observer: $I_I = W\cdot\alpha_I\cdot S_I / (4\pi R_{I\text{-}O}^2)$.
   (We do not take into account effects related to diagrams of scattering and random/mirror scattering. In the student's solutions it's also not really necessary to understand difference between the 2 and the 4 in $2\pi R_{M\text{-}O}^2$ and $4\pi R_{I\text{-}O}^2$.)
   Since Sputnik I was made of highly polished aluminium alloy, we may assume $\cdot\alpha_I = 1$.
   So $I_M / I_I = \alpha_M\cdot(S_M / S_I)\cdot(4\pi R_{I\text{-}O}^2 / 2\pi R_{M\text{-}O}^2) = 2\alpha_M\cdot(d_M/d_I)^2\cdot(R_{I\text{-}O}/R_{M\text{-}O})^2$.
   Necessary data may be taken from the Table of planetary data. Calculations may then give us:
   $I_M / I_I = 0.14\cdot(3475000/0.58)^2\cdot(2.27\cdot10^5/3.72\cdot10^8)^2 = 1.87\cdot10^6$,
   $1.87\cdot10^6 = 1.87\cdot100^3$, i.e. the difference in stellar magnitudes: $0.7 + 3\cdot5 = 15.7$.
   Taking into account that magnitude of the full Moon is -12.7^m, the magnitude of Sputnik I (in zenith at perigee) $m = -12.7^m + 15.7^m = 3^m$. One may see objects of up to magnitude 6^m with the naked eye, so Sputnik may be visible.

2. Large telescopes began to appear only in the beginning of the 20th century. The working surface of one 5-m telescope is equal to the working surfaces of a hundred 50-cm telescopes. There were built only a dozen large telescopes by the end of the 20th century. As an estimate let us assume that the total area of all professional telescopes is 1000 m^2, and that they have observed for a period of 30 years, 365 nights a year, 8 hours a night. This is about 300 000 000 seconds. As usual astronomers do not observe bright objects, especially when using large telescopes, so we take an average magnitude of 1^m to be an upper limit for our estimate. (N.B. It is important for the solution to understand that only light visible by the eye of a researcher, photographic film or CCDs may be used for obtaining knowledge about the structure of the universe.) For flux from Sun, with magnitude -26.8^m, the Total Solar Irradiation Constant is equal to 1.37 kWt/m^2, so the equivalent Total Irradiation Constant for 1^m is equal to $10^{-27.8/2.5}\cdot1.37\ kW/m^2 \approx 10^{-8}\ W/m^2$. So the total energy accumulated by 1000 m^2 during 300 000 000 seconds from a star of 1^m is of order 3000 J, enough only to heat a glass of tea by a few degrees. (In fact, though this need not be taken into account, only about 10% of this energy may be used, since the efficiency of both photographic film and the today's CCDs is still low, a few percents. So more correctly: of order enough to heat a glass of tea for a tenth part of degree.)
   The mass of the water in the swimming pool 50 × 20 × 2 m is 2 000 000 kg or 10^7 glasses. So the estimate of how many degrees it would be possible to the raise temperature of the water in that swimming pool is: not more than of order 10^{-8} K.
   Proposed list of parameters and assumptions used in solution:
   The total square of all professional telescopes is 1000 m^2.
   Period of observing by them is about 300 000 000 seconds.

As usual astronomers observe not bright objects, 1^m to be upper limit for estimation.
For an object of -26.8^m the Total Solar Irradiation Constant may be used.
Not more about 10% of energy may be used due to the low efficiency of CCDs, etc.

3. It seems that since the points are opposite, the situation should be absolutely symmetric: the White Bear should observe sunset at this time. But this is almost not correct. A few days more will pass till the sunset. So the answer "The White Bear observed the sunset" is not correct!
It is necessary to take into account two important circumstances.
First, for any observer, the physical horizon is slightly lowered. For example, if a person stands on a flat surface, the depression of physical horizon she sees is about 2.5'. The depression of physical horizon is easy to calculate: if R is the radius of the Earth, and h is the height of eye level above the surface of the Earth, the horizon dip is equal to

$$\arccos(R/(R+h)) \approx ((R+H)^2 - R^2)^{1/2}/R \approx (2h/R)^{1/2} \text{ (in radians)}$$

Penguins may be different. For an estimation let us take the sizes of an Imperial Penguin, whose eyes' height above the surface of the Earth is about 1 m. The horizon depression is then about 2'. The White Bear, as it is mentioned, is sitting, so height of his eyes above the surface of the Earth will be about 1 m too, and so the horizon depression is about 2' too (If it stands up, this height would be more than 2 m and about 3' accordingly).
This means that if, as seen by the Penguin, the centre of the solar disk is on horizon, this centre already in 2' below mathematical horizon. So at the opposite point of the Earth – at the North Pole – it is 2' above the mathematical horizon. Accordingly, the sitting Bear-observer sees it already in 4' above physical horizon, i.e. a quarter of radius of the Sun.
But horizon depression is actually not the main effect in this problem.
Secondly, there is refraction of light beams in the atmosphere. The value of a refraction depends slightly on weather conditions, but on average contributes about 35' at the horizon. So at the moment when the centre of the solar disk became visible on the horizon at the South Pole, at the opposite point of the Earth – at the North Pole – the Sun has still not begun to set at all. In fact the refraction at the South Pole yields 35', and at North Pole about 25' (not 35', since the Sun not at the horizon but higher). And this effect is the main one in the given problem.
Deduction: the White Bear sees centre of the Sun above the horizon, at approximately 4'+35'+25' = 64' or about one degree.
A picture is necessary to describe this.

Group B

4. If the next opposition of a hypothetical «Mars-2» will be in summer 2018, it means that the synodic period of «Mars-2» is equal approximately to T_S = 15 years. So the sidereal period of «Mars-2» T_{M2} may be found from the formula:

$$1/T_S = 1/T_E - 1/T_{M2},$$

where T_E is sidereal period of Earth,

$$T_{M2} = T_E \cdot T_S / (T_S - T_E) = 1.07 \text{ years.}$$

To find semiaxis a_{M2} of the orbit of «Mars-2» we can use Kepler's 3rd Law:

$$(T_{M2}/T_E)^2 = (a_{M2}/a_E)^3,$$

so

$$a_{M2} = a_E \cdot T_E \cdot T_S / (T_S - T_E)\}^{3/2} = 1.047 \text{ a.u.}$$

Assuming that the orbit of «Mars-2» is circular, the minimum distance to Earth at opposition of this planet will be at 0.047 a.u., i.e. with an apparent size 0.524/0.047 = 11.1 times smaller than in mean opposition of Mars when its magnitude is equal to -2.0^m. «Mars-2» is also closer to Sun with an apparent size 1.524/1.047 = 1.46 times. $5 \cdot \lg(11.1 \times 1.46) = 6.0$, $-2.0^m - 6.0^m = -8.0^m$! So the hypothetical «Mars-2» might be as bright as -8.0^m at opposition.

5. Because Sidereal Time is fixed, the ecliptic orientation is fixed too. Because the Sun moves along ecliptic, the mathematical horizon line coincides with ecliptic at the moment of sunrise (or sunset). This applies only on the Arctic or Antarctic Circles. At what time? It is easy to understand if we take into account that on June 22 on the Arctic Circle the situation is possible only at midnight (at 18^h Local Sidereal Time) or on December 22 only at midday (at 18^h of the Local Sidereal Time too). Analogous considerations give us that on the Antarctic Circle it is possible at 6^h Local Sidereal Time.

Answer: The location can only be on the Arctic Circle at 18^h or on the Antarctic Circle at 6^h Local Sidereal Time.

Note: the term «mathematical horizon» has been used to avoid effects of atmospheric refraction and horizon depression (the main effects in the problem about the White Bear and the Penguin).

6. It is evident that if the anvil impacted the earth after 9 days, the height of the sky is rather great and it is impossible to use the formula $h = gt^2/2$, since acceleration is not constant. The anvil must therefore have followed an elliptical orbit with the center of the Earth at one focus and the apogee of an orbit near to the place, where Hephaestus dropped it. This ellipse must also have been very much prolate: it is hardly likely that Hephaestus gave the anvil a speed of more than a few meters per second by his careless treatment. Thus, we can consider the motion of the anvil for t = 9 days as motion on an almost degenerate ellipse ($e \approx 1$) from the point of apogee up to the surface of the Earth. Apparently the complete period of revolution on such an ellipse is more than 2t = 18 days, comparable to the period of revolution of Moon around Earth ($T_Л$ = 27.3 days); that is, the major semiaxis of our degenerate ellipse is of the same order as distance from the Earth to the Moon ($a_Л$ = 384 thousand km). Even without calculations, then, we can already answer the second question of the problem: the "height of the sky" is comparable to distance to the Moon. But it tells us more: that the size of the Earth may be neglected in comparison with "the height of the sky" h in the solution of our problem. So we may consider that "the height of the sky" h = 2a is the major axis of our ellipse. Using Kepler's 3rd Law, and comparing our orbit with an orbit of Moon we obtain the result

$$(h/2/a_Л)^3 = (2t/T_Л)^2$$

$$h \approx 2 \cdot a_Л \cdot (2t/T_Л)^{2/3} = 1{,}52\ a_Л = 582 \text{ thousand km} \approx 580 \text{ thousand km}$$

or of order 600 thousand km.

It was possible to use information from the “Table of planetary data” for solving of every problem.